Santee Unified School District

MATHEMATICS PROFESSIONAL DEVELOPMENT

Grade One
February 28, 2014

HIGHER-ORDER THINKING & HANDS-ON LEARNING

In a broad-based study (Wenglinsky, 2000) involving over 7,000 students and their teachers, National Assessment of Educational Progress (NAEP) researchers found that, while teacher inputs, professional development, and classroom practices all influence student achievement, the greatest role is played by classroom practices. In particular …

- Math students whose teachers conduct hands-on learning activities outperform their peers by more than 70% of a grade level.

- Math students whose teachers emphasize higher-order thinking skills outperformed their peers by about 40% of a grade level (as compared to lower-order thinking skills – techniques for knowledge acquisition such as memorization or solving problems that are similar to one another and often referred to as drill and practice or learning by rote).

RESEARCH ON HIGHER-ORDER THINKING

In a well-known synthesis study for the National Research Council (Resnick, 1987), higher-order thinking skills are defined as: elaboration of material presented; drawing inferences; analysis; and the construction of relationships. Further, the study characterizes higher-order thinking as:

1. "…**non-algorithmic** – it is not routine; the path of action is not fully specified in advance.

2. **…complex** – the total path is not "visible" (mentally speaking) from any single vantage point.

3. …yielding **multiple solutions**, each with costs and benefits, rather than unique solutions.

4. …involving **nuanced judgment** and interpretation.

5. …involving the application of **multiple criteria** which sometimes conflict with one another.

6. …**uncertain** – not everything that is relevant to the task at hand is known.

7. …requiring **self-regulation** of the thinking process – this does not occur when someone else tells you what to do at every step.

8. …**imposing meaning** or **finding structure** in apparent disorder.

9. …**effortful** – there is considerable mental work involved in the kinds of elaborations and judgments required."

In yet another study (Resnick et al, 1991), researchers found that young children and low-performing students can learn and use the same **reasoning strategies and higher-order thinking skills** that are used by high-performing students.

Wenglinsky, H. *How teaching matters: Bringing the classroom back into discussions of teacher quality,* Princeton, NJ: The Milken Family Foundation and Educational Testing Service, October, 2000.

Resnick, L. B., *Education and Learning to Think*, Washington, D.C.: National Academy Press, 1987.

Resnick, L.B., et. al. Thinking in the arithmetic class. In B. Means, et. al. (Eds.), *Teaching advanced skills to at-risk students: views from research and practice* (pg. 27-53). San Francisco: Jossey-Bass, 1991.

JULIE – WORTHWHILE ACTIVITIES

“In order for a math class to be worthwhile, I believe that the teacher needs to provide the students with worthwhile activities. For example, when we explain our thinking out loud, make posters of our work, draw diagrams, work in groups, and use manipulatives, we are more productive and therefore learn more. … To determine if an activity is worthwhile, some helpful things to ask are:

- Will it make students stretch their thinking?
- Will it branch off to other topics?
- Is there more than one way to solve the problem?
- Will it help students’ understanding of the idea?
- Will it cause some disequilibrium?

… The best activities to do are ones for which the teacher can say ‘yes’ to all of the above questions. Teachers can’t expect the students to work well in groups if they give them a problem like, “Find the sum of twenty-eight and seventeen.” Better questions to ask would be, ‘Can you find more than one way to solve the problem twenty-eight plus seventeen?’ or ‘Can you invent an algorithm for adding any two digit numbers?’ …”

Julie

MARTHA'S CARPETING TASK

Martha was re-carpeting her bedroom, which was 15 feet long and 10 feet wide. How many square feet of carpeting will she need to purchase?

FENCING TASK

Ms. Brown's class will raise rabbits for their spring science fair. They have 24 feet of fencing with which to build a rectangular rabbit pen to keep the rabbits.

1. If Ms. Brown's students want their rabbits to have as much room as possible, how long would each of the sides of the pen be?

2. How long would each of the sides of the pen be if they had only 16 feet of fencing?

3. How would you go about determining the pen with the most room for any amount of fencing? Organize your work so that someone else who reads it will understand it.

The Characteristics of Mathematical Tasks at Each of the Four Levels of Cognitive Demand

LOWER-LEVEL DEMANDS	*HIGHER-LEVEL DEMANDS*
Memorization Tasks • Involve either reproducing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memeory. • Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.	**Procedures With Connections Tasks** • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • Usually are represented in multiple ways (e.g. visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
Procedures Without Connections Tasks • Are algorithmic. Use of procedures is either specifically called for or its use is evident based on prior instruction, experience, or placement of the tasks. • Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • Have no connection to the concepts or meaning that underlie the procedure being used. • Are focused on producing correct answers rather than developing mathematical understanding. • Require no explanations, or explanations that focus solely on describing the procedure that was used.	**Doing Mathematics Tasks** • Require complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). • Require students to explore and understand the nature of mathematical concepts, processes, or relationships. • Demand self-monitoring or self-regulation of one's own cognitive processes. • Require students to assess relevant knowledge and experiences and make appropriate use of them in working through the task. • Require stuents to analyze the task and actively examine task constraints that may limite possible solution strategies and solutions. • Require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Source: Stein, Smith, Henningsen and Silver, 2009 *Implementing Standards Based Mathematics Instruction*. NCTM/Teachers College Press

The Characteristics of Mathematical Tasks at Each of the Four Levels of Cognitive Demand

LOWER-LEVEL DEMANDS

Memorization Tasks

What are the decimal and percent equivalents for the fractions ½ and ¼?

Expected student response:

½ = 0.5 = 50%
¼ = 0.25 = 25%

Procedures Without Connections Tasks

Convert the fraction 3/8 to a decimal and a percent

Expected student response:

Fraction

3/8

Decimal

```
   0.375
8) 3.00
   24
    60
    56
     40
     40
```

Percent

0.375 = 37.5%
(move decimal two places to the right)

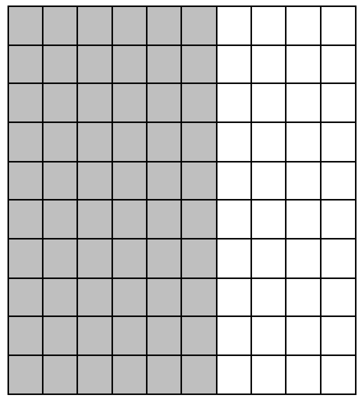
HIGHER-LEVEL DEMANDS

Procedures With Connections Tasks

Using a 10 x 10 grid, identify the decimal and percent equivalents of 3/5.

Expected student response:

Pictorial



Fraction
60/100 = 3/5

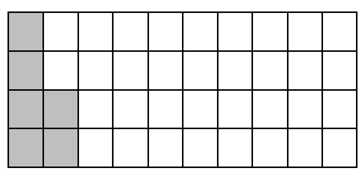
Decimal
60/100 l= 0.60

Percent
0.60 = 60%

Doing Mathematics Tasks

Shade 6 small squares in a 4 x 10 rectangle. Using the rectangle, explain how to determine each of the following: (a) the percent of area that is shaded, (b) the decimal part of area that is shaded, and (c) the fractional part of area that is shaded.

One possible student response



(a) One column will be 10%m since there are 10 columns. So, four squares is 10%. Then 2 squares is half a column and half of 10%, which is 5%. So the 6 shaded blocks equal 10% plus 5%, or 15%.

(b) One column will be 0.10, since there are 10 columns. The second column has only 2 squares shaded, so that would be one-half of 0.10, which is 0.05. So the 6 shaded blocks equal 0.1 plus 0.05, which equal 0.15.

(c) Six shaded squares out of 40 squares is 6/40 , which reduces to 3/20.

Source: Stein, and Smith, January, 1998, *Mathematical Tasks as a Framework for Reflection: From Research to Practice,* Mathematical Teaching in the Middle School, Vol. 3, No. 4.

WORTHWHILE TASKS

Enhance the following tasks to increase the likelihood of a "yes" on Julie's list* of helpful things to ask. Try not to change the intent of the task.

**To determine if an activity is worthwhile, some helpful things to ask are:*

- *Will it make students stretch their thinking?*
- *Will it branch off to other topics?*
- *Is there more than one way to solve the problem?*
- *Will it help students' understanding of the idea?*
- *Will it cause some disequilibrium?*

Task	Enhanced Task
1. The number is 82. How many tens?	
2. What number follows 45?	
3. Mrs. Farmer had 9 chickens. She sold 2 of them. How many chickens does she have left?	
4. You have one dime, two nickels and two pennies. How much money do you have?	
5. 3+4 = ☐; 5 + 2 = ☐; 6 + 1 = ☐	
6. Count to 20 by twos.	
7. Add. Show your work 28 <u>+ 17</u>	

Student Discourse Observation Tool

Procedures/Facts	Justification	Generalization
• Short answer to a direct question • Restating facts/statements made by others • Showing work/methods to others • Explaining what and how • Questioning to clarify • Making observations/connections	• Explaining why by providing mathematical reasoning • Challenging the validity of an idea by providing mathematical reasoning • Giving mathematical defense for an idea that was challenged	Using mathematical relationships as the basis for: • Making conjectures/predictions about what might happen in the general case or in different contexts • Explaining and justifying what will happen in the general case

Scripting of Student Discourse	Discourse Type P/F, J, or G

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Classroom Observation - Reflection

1. What mathematical ideas did students seem to understand? What is your evidence?

2. With what mathematical ideas were students struggling? What is your evidence?

3. How would you characterize the students' mathematical discourse?